Multiple-point statistical approach to model geological heterogeneity

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What is multiple-point statistics?

It is an alternate approach to categorical geostatistics that uses instead of a variogram a user given training image to estimate the conditional probability at interpolation location given the observed and the already interpolated data (Boogaart, 2006)
## Comparison

<table>
<thead>
<tr>
<th>Measure</th>
<th>Two-point statistics</th>
<th>↔</th>
<th>multiple-point statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measure</strong></td>
<td>Statistical relation of two points $z(u)$ and $z(u+h)$</td>
<td></td>
<td>Structures and patterns beyond 2-point correlation</td>
</tr>
<tr>
<td><strong>Conditional probability</strong></td>
<td>Variogram model</td>
<td></td>
<td>Training image</td>
</tr>
<tr>
<td><strong>Parameterization</strong></td>
<td>Sill, Range, Nugget</td>
<td></td>
<td>Proportions, Scale, Anisotropy</td>
</tr>
<tr>
<td></td>
<td>Shape of Variogram</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Training image

Conceptual drawing

Arbitrary picture

Real picture
Parameterization

Proportions (weights): 1:0.7, 1:1, 1:1.4

Scales: rot: 0, rot:45, isotropy

Anisotropy
Multiple-point statistics (MPS) algorithms

ref: Strebelle, 2002

Categorization of variables:

C(x) = 1 (Clay)
C(x) = 2 (Sand)

Sequential simulation:

• choose unknown location
• find closest nodes
• find C(x)
Algorithm II

```
<table>
<thead>
<tr>
<th>Event</th>
<th>Counter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>
```

data event

- Clay
- Sand
Algorithm III

$C(x)$ is drawn from the probability distribution (pdf)

Continuation:

- Select and simulate another random location
- Complete the full model domain
Simulation

Braided river channel

MPS simulation

Meanders

MPS simulation

ref: Renard, 2009
What steps are required for a field application:

1. Collection of training images
2. Identification and categorization of lithofacies
3. Simulation of spatial variability
4. Conditioning of the simulations to a well field/contaminated site
Vadsby Site
Field Investigation

Upper clay till unit

Transition zones with sand lenses

Meltwater deposits
Sand lenses

Sand sheets

Sand stringers

Sand pockets
Defining a cross-section
Map the sand lenses
Categorize the lithofacies clay till sand lenses
Simulation

SGeMS, developed by Remy et al. (2008)
Calculation of geobodies

Validation
• collect 3D training images in the field
  → in many cases not feasible

• generate 3D training images based on geological knowledge on shapes and geometry of sand lenses

• build a 3D training images based on simulations
  → condition simulation to themselves
(3D) Training Image
Simulation path

- Training Images
- Simulation slices
- Conditioning point
Simulation results
Simulation results

Impala, developed by Straubhaar and Renard (2009)
Conclusions

Multiple-point simulation

- multiple-point statistics simulate complex shapes and geometries
- 3D simulations are enabled with sequential data conditioning techniques
- Validation of simulation only possible with numerous repetition of realizations

- SGemS provides a GUI for simulating mps
- only 2D training images can be processed
The end